NOTES ON ESTIMATING RUN-OFF

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The design of works for control of water requires the estimate of future run-off. This is usually done by taking the probability of occurrence a priori equal to that a posteriori, which procedure often based on short records does not take into consideration the oscillations in precipitation which occur. Longer records do not entirely

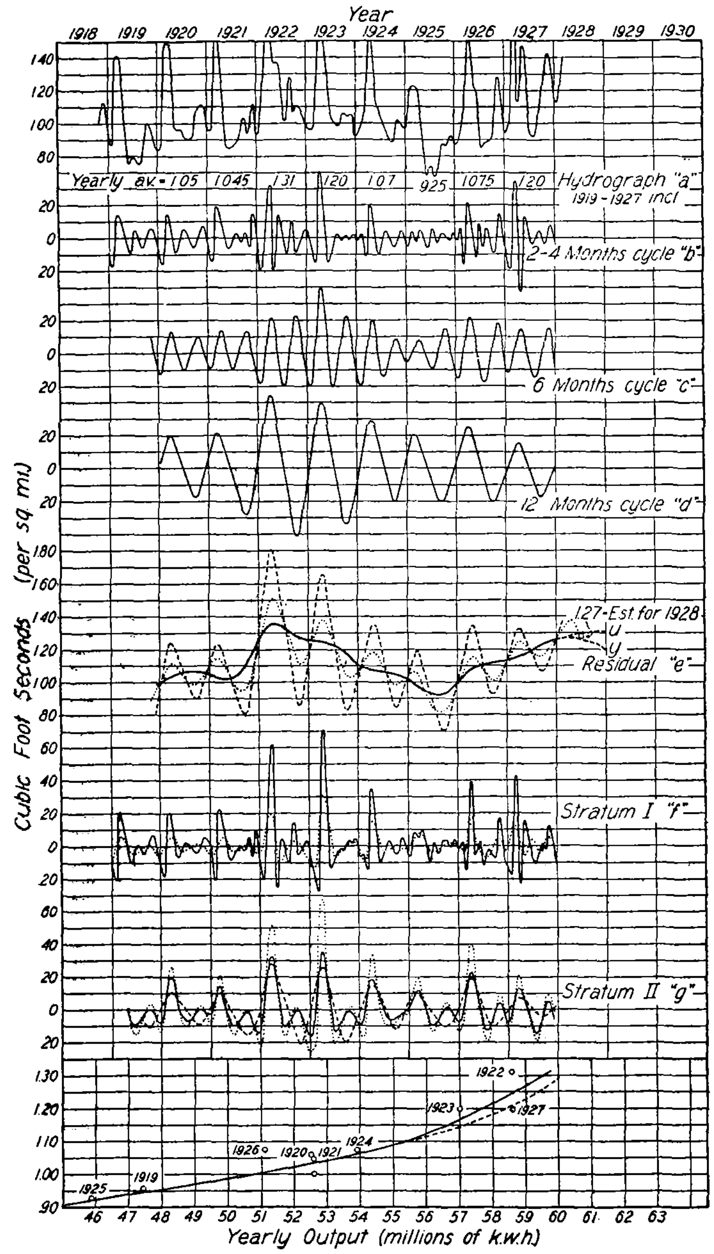

FIG. 1

solve the problem for the existence of long-term trends in rainfall and hence run-off is an established fact. (1) A better procedure is to analyze these trends and oscillations and to estimate their extrapolation for such length of time as may be involved in the project under consideration. Covering shorter periods such estimates have their value for operating purposes. For hydroelectric plants the estimate of next year's output is of value. The water output is of value. The water supply of large cities and the operation of irrigation reservoirs require an estimate of the balance of water receivable into and to be drawn from storage in order to anticipate remedial measures in case of expected depletion.

Precipitation and run-off are, in engineering, usually regarded as chance phenomena. Adoption of such a fortuity theory renders a solution of above problem impossible. It is, moreover, not entirely true. While indeed the short-term variations exhibit some fortuitous characteristics, typical oscillations and trends of longer duration may be isolated from the observations. It is remarkable that run-off, notwithstanding its dependence on a multitude of variable factors such as transpiration, seepage, evaporation, topography and geology, exhibits more regular features than rain-gage data, probably on account of the equalizing effect of large drainage areas and ground storage.

In previous study (2) a method was given suitable for estimating run-off. Application on the Manistee River in Michigan is given in Figure 1. The Manistee River at the point selected has a drainage area of 1,451 square miles It originates on the high plateau of glacial till and overwash which covers the Lower Michigan Peninsula 600 feet deep, and empties into Lake Michigan. In the 12,000 years since retreat of the last ice sheet (3) the river has excavated a deep valley in this glacial till which forms a vast underground storage reservoir resulting in exceptionally even flow (fig. 1) with a maximum range of variation of the yearly average of ± 30 per cent of the mean, which is smaller than usual.

In order to render the hydrograph *a* (fig. 1) suitable for extrapolation, it must be reduced to an equivalent curve of simpler form *e* (fig. 1). It appears that this is possible in the following manner: The monthly averages are added four times and the result reduced to phase and scale dividing by 16 and displacing 2 months. (See Table 1.) The result is now subtracted from column 1 and gives the first stratum, *f*. Then, the first residual (column 5) is again four times added with an interval of 2 months. Thus, if the monthly figures are *a*, *b*, *c*, *d*, *e*, these sums are formed $a+c$, $b+d$, $c+e$, etc. This again is repeated four times. The result is again divided by 16 and displaced 4 months in order to reduce to correct scale and phase. (Column 10.) Subtracting column 10 from column 5 gives the second stratum, *g*. An example of the summation is given in Table 1; it may be carried on until the end of the record.

A median line can now be drawn through the oscillations of the first stratum, and it may be seen that this median line in appearance follows the outline of stratum *g*. The difference between this median line and the first stratum, plotted separately, gives cycle *b*.

This median line is now added to the second stratum *g* and a median line drawn through this corrected curve. It may be seen that this median line in appearance again follows the outline of the remaining residual and it is added to this. The difference, with the second stratum separately plotted, gives cycle *c*.

A median line is now drawn through residual *e*, and the oscillations above and below this line separately plotted. This gives cycle *d*.

The original hydrograph is equal to the sum of the 4 elements *b*, *c*, *d*, and *e*. We have now the following remarkable result:

Averaging residual e year by year, we obtain within a few per cent the average yearly flow of the river. The hydrograph is therefore equivalent to residual e in giving the average annual flow.

In other words, the sum of elements b, c and d over each year is practically equal to zero.

A closer inspection of b, c and d shows that d is an annual fluctuation variable in amplitude, but with a constant period of one year, c is a six month cycle, also of variable amplitude and constant period. C and d are so regular in appearance that there can not be any suggestion of fortuity. These oscillations are apparently seasonal.

Cycle b is more irregular oscillation and suggests presence of a fortuitous element. The annual peaks vary from three to five in number. Nevertheless, it offers some regular features. A maximum occurs for instance, in April each year except in the lowest year 1925. It is this cycle which precludes estimate of run-off month by month, although taken over a whole year, the sum is practically zero.

These three cycles are therefore seasonal fluctuations around a mean value expressed in residual e. They do not affect the mean flow of the year.

The hydrograph is now reduced to an equivalent residual e. This residual is of such simple appearance that it may be extrapolated with a certain degree of probability. The following points are here to be observed:

The record, if extending over a whole number of years, is shortened by repeated addition, so that residual c does not extend to the end of the year. It is easily extended because the mean flow for the last year of the record is known and must be equal to the mean of element e over that year.

We know, therefore, the starting point of residual e for the year to come. The extension is aided by several considerations. To begin with, a relation with the Wolf numbers may be established, which for some regions, as this one, is very pronounced. Two maxima appear in an 11-year period, one during minimum and another during sunspot maximum. Also, the observation of Doctor Bauer 4 that it is the rate of change rather than the absolute magnitude of the Wolf numbers which is the important factor, seems to apply to run-off in this region. The maxima in run-off leads the maxima in Wolf numbers somewhat. Hence, we may expect that the extension e for 1928 is not materially higher than 1927. Likewise a low flow in 1928 can not be expected for the same reason and the extension of e may therefore perhaps follow u or y. A further consideration favors both. Superimposed on e are oscillations with a variable period which are about 1.5–1.6 years apart and which should culminate again in the beginning of 1928. Then the annual cycle gives an indication of the extension of c. The minimum in 1927 is located at such a point, that the maximum in 1928 can not be very high (point o), and hence it is unlikely that e will rise abruptly as the amplitude of point o over e is already as small as occurred through the record. On the other hand, the relation to the Wolf numbers precludes the probability of an abrupt decline.

These considerations allow an extension of e with, as previously found, considerable degree of probability, and therewith an estimate of the mean flow for the following year. It may be seen that the extensions may be varied in a considerable range without varying the mean value more than 5 per cent.

The above considerations are based on the existence of a continuity in the mean annual values of run-off as disclosed by residual e. Adopting a fortuitous sequence such estimates as the above become impossible. The estimate has no practical value as an interpretation of next year's rainfall, for the seasonal distribution thereof is, of course, the issue which determines its practical importance. But, for all works involving annual storage of water, the estimate is of value. While of proven reliability in the wet regions its applicability to semi-arid regions has not been investigated.

LITERATURE CITED

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2. Monthly Weather Review, November, 1926.
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TABLE 1.—*Residuation of Manistee River hydrograph*

Mo. av. run-off c. f. s/sq. m.		1	2	3	4	5	6	7	8	9	10	1–5	5–10
1918												Stratum f	Stratum g
November	1.01	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----		
December	1.13	2.14	-----	-----	-----	-----	-----	-----	-----	-----	-----		
1919													
January	.95	2.08	4.22	-----	-----	1.01	-----	-----	-----	-----	-----	−0.06	-----
February	.86	1.81	3.89	8.11	-----	1.07	-----	-----	-----	-----	-----	−.21	-----
March	1.42	2.28	4.09	7.98	16.09	1.21	2.22	-----	-----	-----	-----	+.21	-----
April	1.34	2.76	5.04	9.13	17.11	1.22	2.29	-----	-----	-----	-----	+.12	-----
May	1.04	2.38	5.14	10.18	19.31	1.05	2.26	4.48	-----	-----	1.01	−.01	+0.04
June	.75	1.79	4.17	9.31	19.49	.87	2.09	4.38	-----	-----	.95	−.12	−.08
July	.79	1.54	3.33	7.50	16.81	.79	1.84	4.10	8.58	-----	.89	.00	−.10
August	.77	1.56	3.10	6.43	13.93	.78	1.65	3.74	8.12	-----	.86	−.01	−.08
September	.75	1.52	3.08	6.18	12.61	.81	1.60	3.44	7.54	16.12	.86	−.06	−.05
October	.88	1.63	3.15	6.23	12.41	.88	1.66	3.31	7.05	15.17	.88	.00	.00
November	1.01	1.89	3.52	6.07	12.90	.93	1.74	3.34	6.78	14.32	.91	+.08	+.02
December	.94	1.95	3.84	7.36	14.03	.92	1.80	3.46	6.77	13.82	.96	+.02	−.04
1920													
January	.83	1.77	3.72	7.56	14.92	.92	1.85	3.59	6.93	18.71	1.02	−.09	−.10
February	.86	1.69	3.46	7.18	14.94	1.05	1.97	3.77	7.23	14.00	1.07	−.19	−.02
March	1.47	2.33	4.02	7.48	14.66	1.26	2.18	4.03	7.62	14.55	1.11	+.21	+.15
April	1.48	2.95	5.28	9.30	16.78	1.31	2.32	4.33	8.10	15.23	1.12	+.17	+.19
May	1.09	2.57	5.52	10.80	20.10	1.17	2.43	4.61	8.64	16.26	1.11	−.08	+.06
June	.95	2.04	4.61	10.13	20.93	1.02	2.33	4.69	9.02	17.12	1.06	−.07	−.04
July	.97	1.92	3.96	8.57	18.70	.95	2.12	4.55	9.16	17.80	1.02	+.02	−.07
August	.90	1.87	3.79	7.75	16.32	.92	1.94	4.27	8.96	17.98	.99	−.02	−.07
September	.90	1.80	3.67	7.46	15.21	.92	1.87	3.99	8.54	17.70	.98	−.02	−.06
October	.91	1.81	3.62	7.29	14.75	.96	1.88	3.82	8.09	17.05	.99	−.05	−.03
November	1.07	1.98	3.80	7.42	14.71	1.03	1.95	3.82	7.81	16.35	1.03	+.04	.00
December	1.12	2.19	4.17	7.97	15.39	1.07	2.03	3.91	7.73	15.82	1.07	+.05	.00